Localization of Unknown Signals over Multipath Channels

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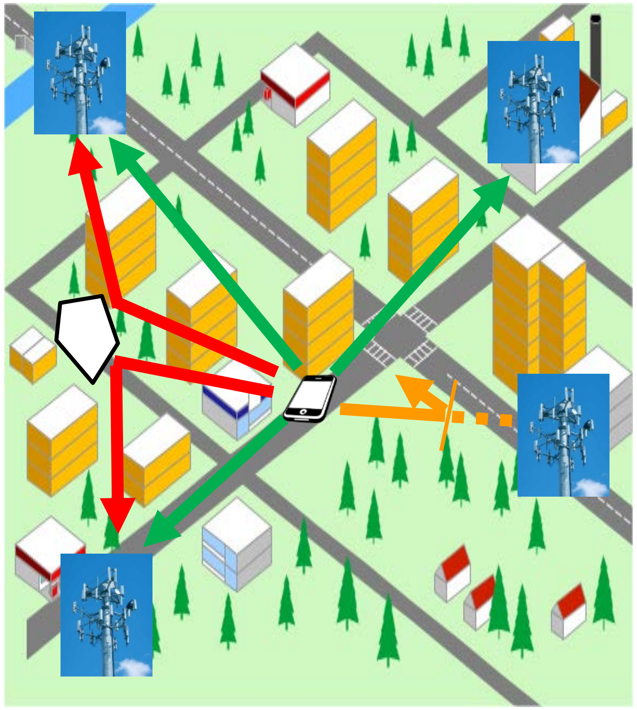
At a Glance

- Goal: Localization (geolocation) of <u>unknown</u> RF emitters in multipath environments

- Challenges:
  - Conventional methods such as TDOA based on line-of-sight (LOS)
  - Non-line-of-sight (NLOS) paths
  - Blocked LOS paths (e.g. indoor source)

- Applications:
  - Defense applications
  - Location based services
  - E911



Problem Statement

Goal

- Estimate emitters locations

Sensor assumptions

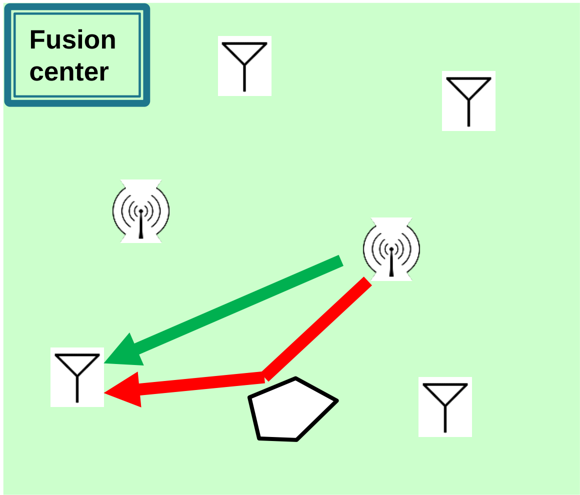
- Network of distributed sensors with fixed, known locations
- Sensors have ideal communication with a fusion center
- Sensors are time synchronized

Source assumptions

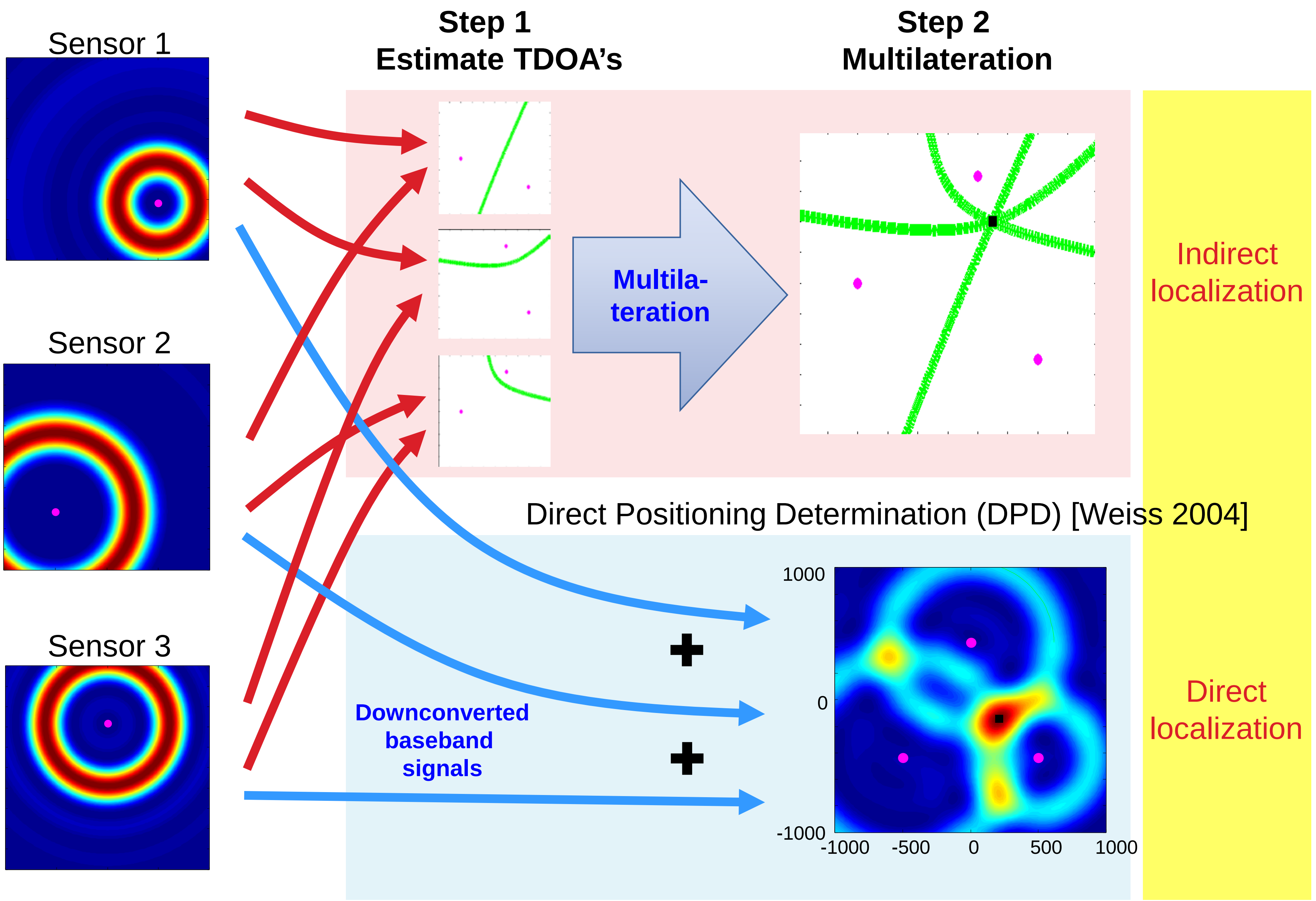
- Emitter waveforms are unknown
- PSD is known

Channel assumptions

- Time-invariant unknown multipath channel
- No prior information on the multipath channel

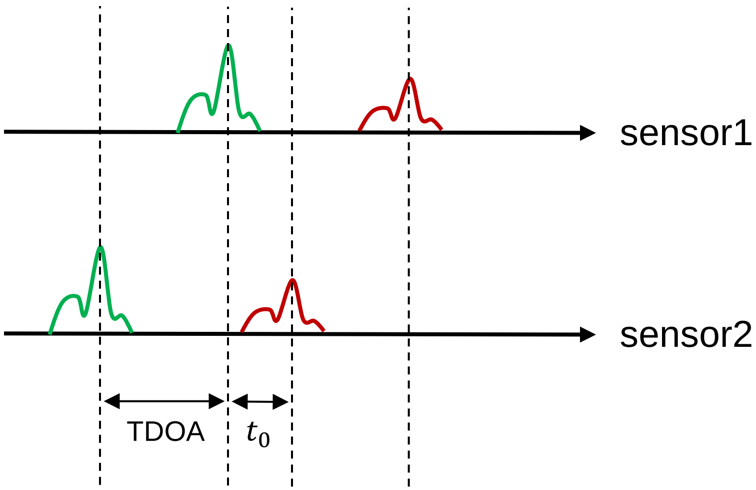


Localization over LOS Channels

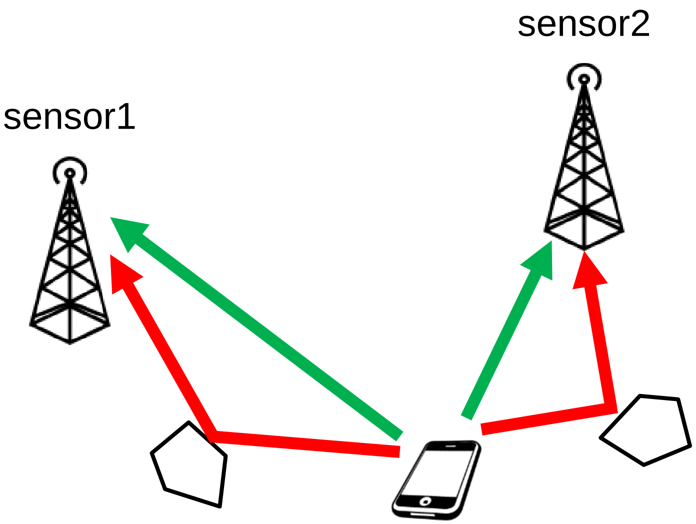


Multipath Challenge

- TDOA, DPD fail in multipath channels – can not apply the principle that the shortest delay difference = LOS TDOA
- Very scarce literature on localizing emitters over multipath channels.



$$t_0 < \text{TDOA}$$



Signal Model

- Signal at n-th sensor

$$\tilde{z}_n(t) = \underbrace{\sum_{q=1}^{Q} \alpha_{nq} \tilde{s}_q\left(t - \tau_n\left(\mathbf{p}_q\right)\right)}_{\text{LOS}} + \underbrace{\sum_{q=1}^{Q} \sum_{m=1}^{M_{nq}} \beta_{nq}^{(m)} \tilde{s}_q\left(t - \tau_{nq}^{(m)}\right)}_{\text{NLOS}} + \tilde{w}_n(t)$$

LOS **NLOS**

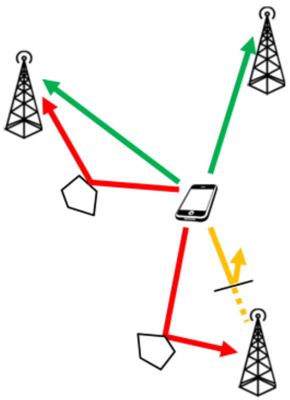
- N sensors, Q emitters
- $\tilde{s}_q =$ source signal
- LOS

  $\alpha_{nq} =$ complex gain LOS path

  $\mathbf{p}_q =$ source location
- NLOS

  $\beta_{nq}^{(m)} =$ complex gain NLOS path

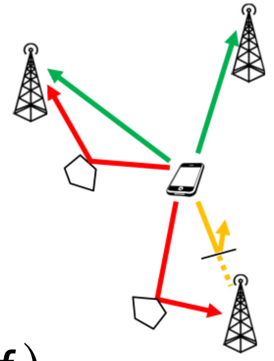
  $\tau_{nq}^{(m)} =$ multipath time delay



Signal model for n-th sensor (frequency domain)

$$z_n(f) = \sum_{q=1}^{Q} \alpha_{nq} s_q(f) e^{-j2\pi f \tau_n(\mathbf{p}_q)} + \sum_{q=1}^{Q} \sum_{m=1}^{M_{nq}} \beta_{nq}^{(m)} s_q(f) e^{-j2\pi f \tau_{nq}^{(m)}} + w_n(f)$$

LOS **NLOS**



Covariance matrix

$$\mathbf{R}(f) = E\left[\mathbf{z}(f)\mathbf{z}^H(f)\right]$$

Model of vectorized covariance matrix $\gamma(f) = \text{vec}(\mathbf{R}(f))$

$$\gamma_n(f) = \sum_{q=1}^{Q} x_{nq} S_q(f) e^{-j2\pi f \tau_n(\mathbf{p}_q)} + \sum_{q=1}^{Q} \sum_{m=1}^{M_{nq}} y_{nq}^{(m)} S_q(f) e^{-j2\pi f \tau_{nq}^{(m)}} + w_n(f)$$

LOS **NLOS**

$S_q(f) =$ known power spectral density

Covariance matrix depends on unknown parameters $\gamma(f;\boldsymbol{\theta})$

$$\boldsymbol{\theta} = \left[x_{nq}, p_q, M_{nq}, y_{nq}^{(m)}, \tau_{nq}^{(m)}\right]$$

Covariance Matching Estimation Technique (COMET)

- MLE of unknown parameters from covariance matrix is too difficult
- Simpler alternative: approximate cov mat with sample cov mat

Compute the sample covariance matrix

$$\hat{\mathbf{R}} = \frac{1}{J}\sum_{j=1}^{J}\mathbf{z}_j\mathbf{z}_j^H$$

Vectorize

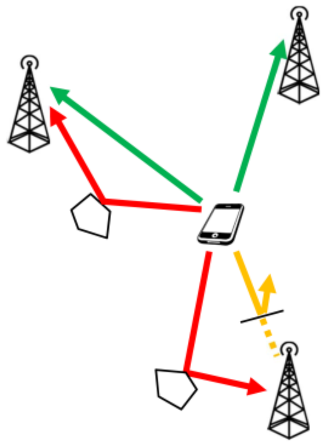
$$\hat{\gamma} = \mathrm{vec}\left(\hat{\mathbf{R}}\right)$$

Solve optimization to find $\boldsymbol{\theta}$

$$\min_{\boldsymbol{\theta}}\left(\hat{\gamma} - \gamma(\boldsymbol{\theta})\right)^H \mathbf{C}^{-1}\left(\hat{\gamma} - \gamma(\boldsymbol{\theta})\right)$$

C is the covariance matrix of the residue

$$\mathbf{C} = E\left[\left(\hat{\gamma} - \gamma\right)\left(\hat{\gamma} - \gamma\right)^H\right] \approx \frac{1}{J}\left(\hat{\mathbf{R}} \otimes \hat{\mathbf{R}}\right)$$



Localization from SCM

Even applying COMET leads to very complex problem

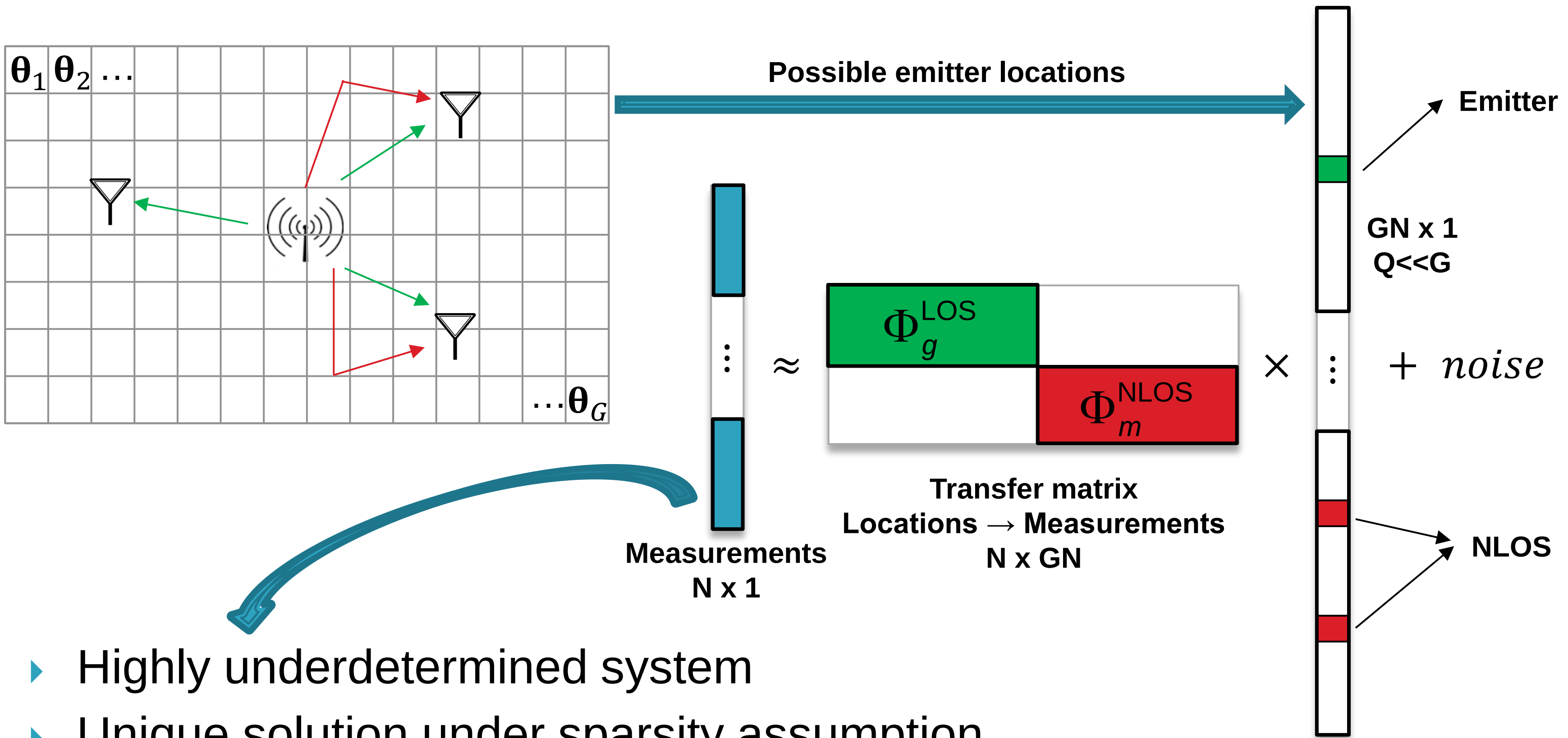
- Measurements
- Unknown parameters related to LOS paths
- Unknown parameters related to NLOS paths

$$\min_{\substack{x_{nq}, p_q \\ M_{nq} \\ y_{nq}^{(m)}, \tau_{nq}^{(m)}}} \left\| \mathbf{C}^{-1/2} \left(\hat{\gamma} - \gamma \left(x_{nq}, p_q, M_{nq}, y_{nq}^{(m)}, \tau_{nq}^{(m)} \right) \right) \right\|^2$$

- Large pool of unknown parameters
- Impractical complexity

Sparsity

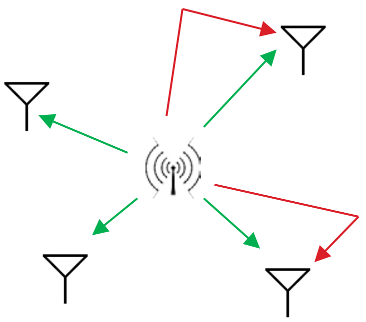
1. A small number of sources Q to be localized
2. A large number of possible locations for the sources G >> Q



- Highly underdetermined system
- Unique solution under sparsity assumption
- Efficient algorithms – active area of research

Proposed Localization Method

- Emitters are sparse
- LOS originate from common location
- NLOS is local to the sensors
- Mixed norm optimization to control LOS vs. NLOS assignments



$$\min_{\mathbf{x}_g, \mathbf{y}_m} \sum_{g=1}^{G} v \left\| \mathbf{x}_g \right\|_2 + \sum_{m=1}^{M} \left\| \mathbf{y}_m \right\|_1$$

$$\text{subject to } \left\| \mathbf{C}^{-1/2} \left(\hat{\gamma} - \gamma \right) \right\|_2^2 \leq \varepsilon$$

$$\gamma = \sum_{g=1}^{G} \Phi_g^{\text{LOS}} \mathbf{x}_g + \sum_{m=1}^{M} \Phi_m^{\text{NLOS}} \mathbf{y}_m$$

Design challenge

- Explain received data with correct mixture of LOS and NLOS

$$\min_{\mathbf{x}_g, \mathbf{y}_m} \sum_{g=1}^{G} v \left\| \mathbf{x}_g \right\|_2 + \sum_{m=1}^{M} \left\| \mathbf{y}_m \right\|_1$$

- Source is missed when LOS is explained as NLOS
- False alarm occurs when NLOS is explained as LOS

Theorem

- Given measurements collected by L sensors and $L_1 < L$, if $\sqrt{L_1(L_1-1)-1} < v < \sqrt{L_1(L_1-1)}$, then the optimization problem will seek a feasible solution that explains a source with no less than L_1 LOS components

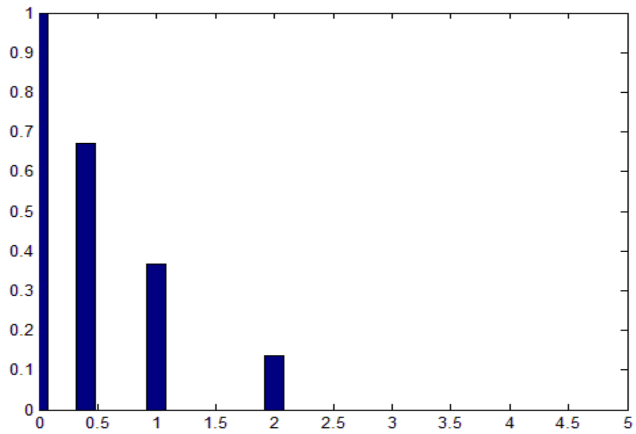
Significance

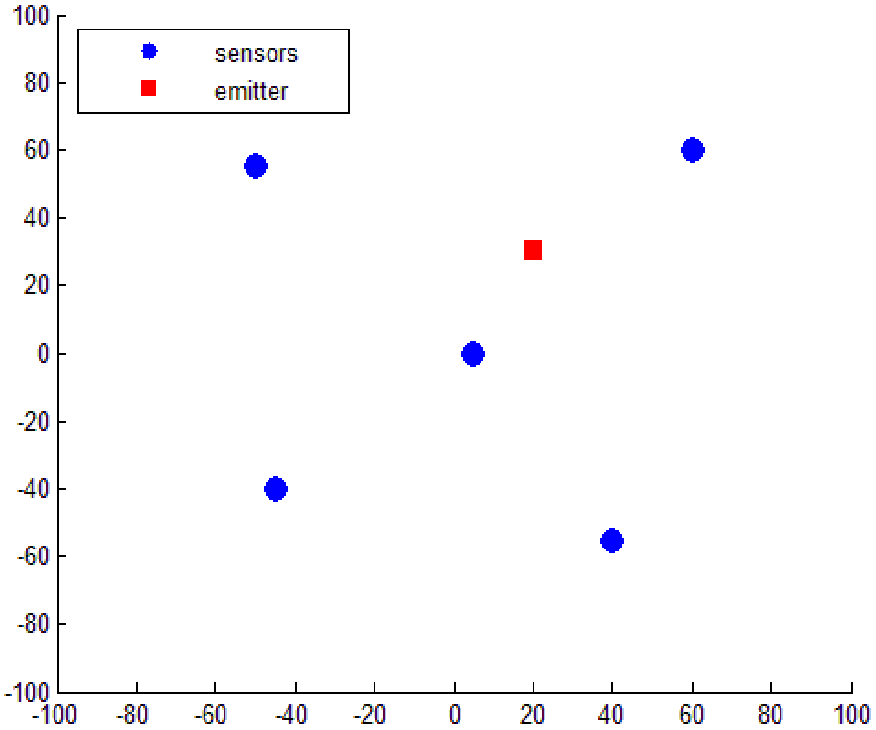
- L_1 LOS components are not explained as NLOS (prevent missed source)
- Fewer than L_1 NLOS components are not explained as LOS (prevent false alarm)

Numerical Results

- 10 MHz emitter (30 m ranging resolution)
- Multipath channel RMS delay spread is 500 ns (exponential profile, Poisson arrivals)
- Search area: 200 x 200 m
- 5 sensor and 1 emitter
- 1000 samples/sensor

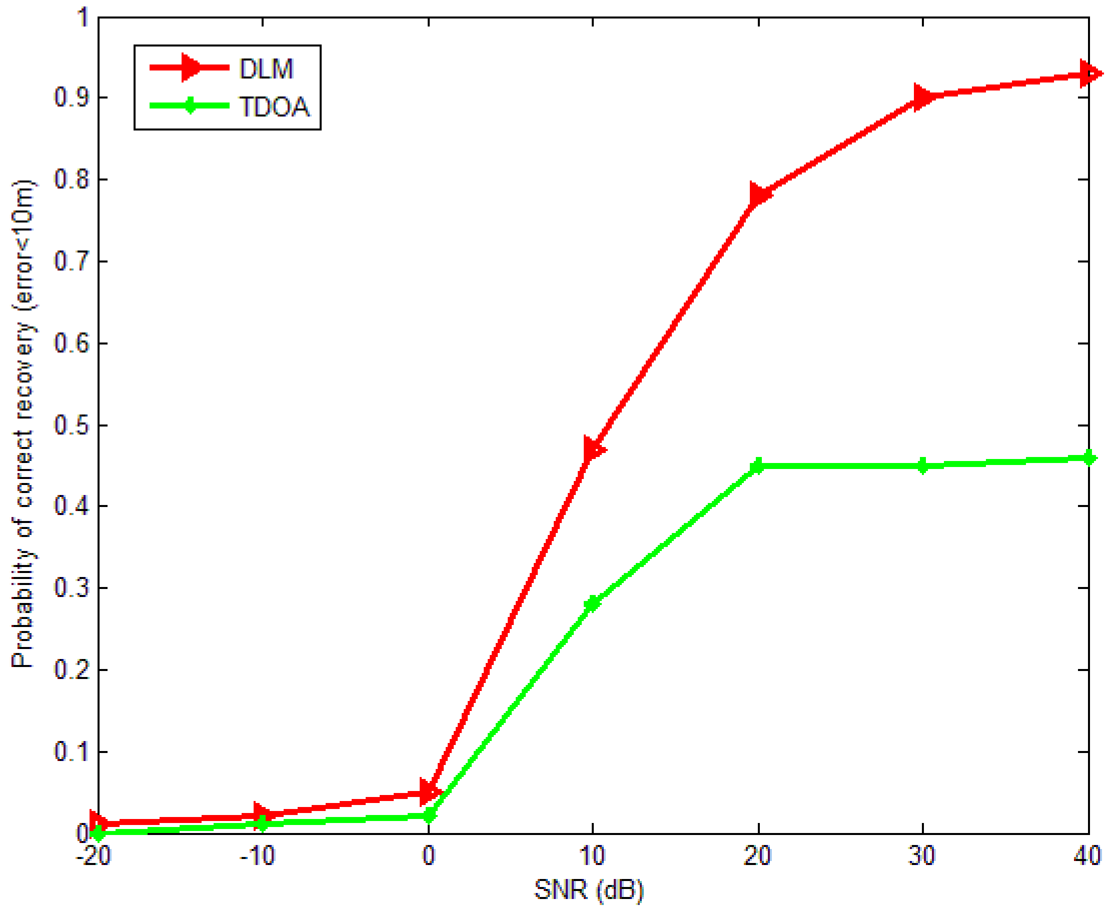
Channel impulse response





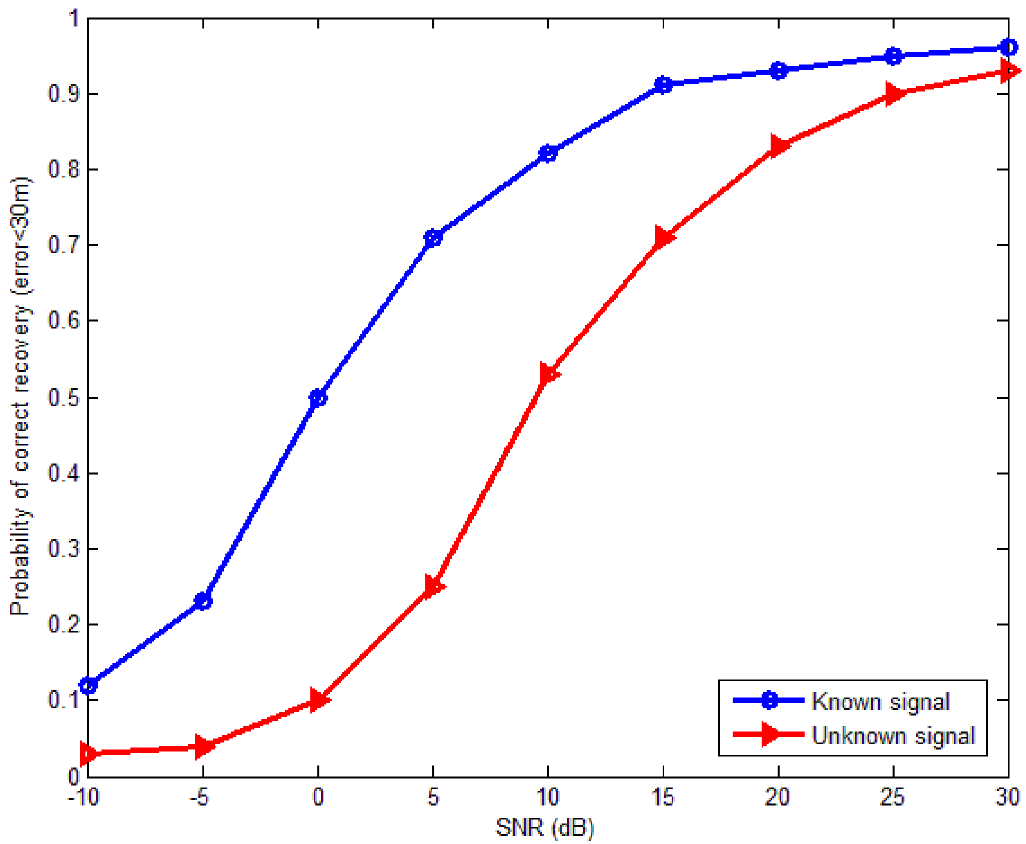
Probability of Correct Recovery vs. SNR

- Correct recovery if error smaller than 10 m
- Unknown signal



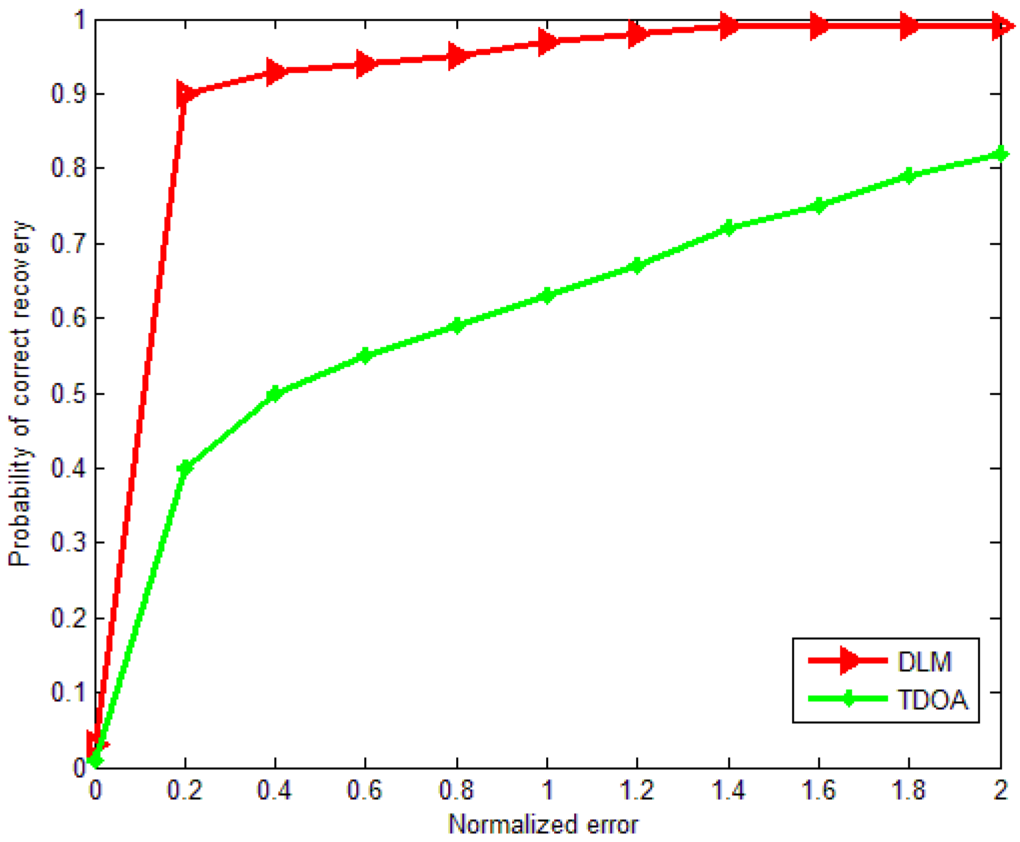
Unknown Signals Vs. Known Signals

- Correct recovery if error smaller than 30 m



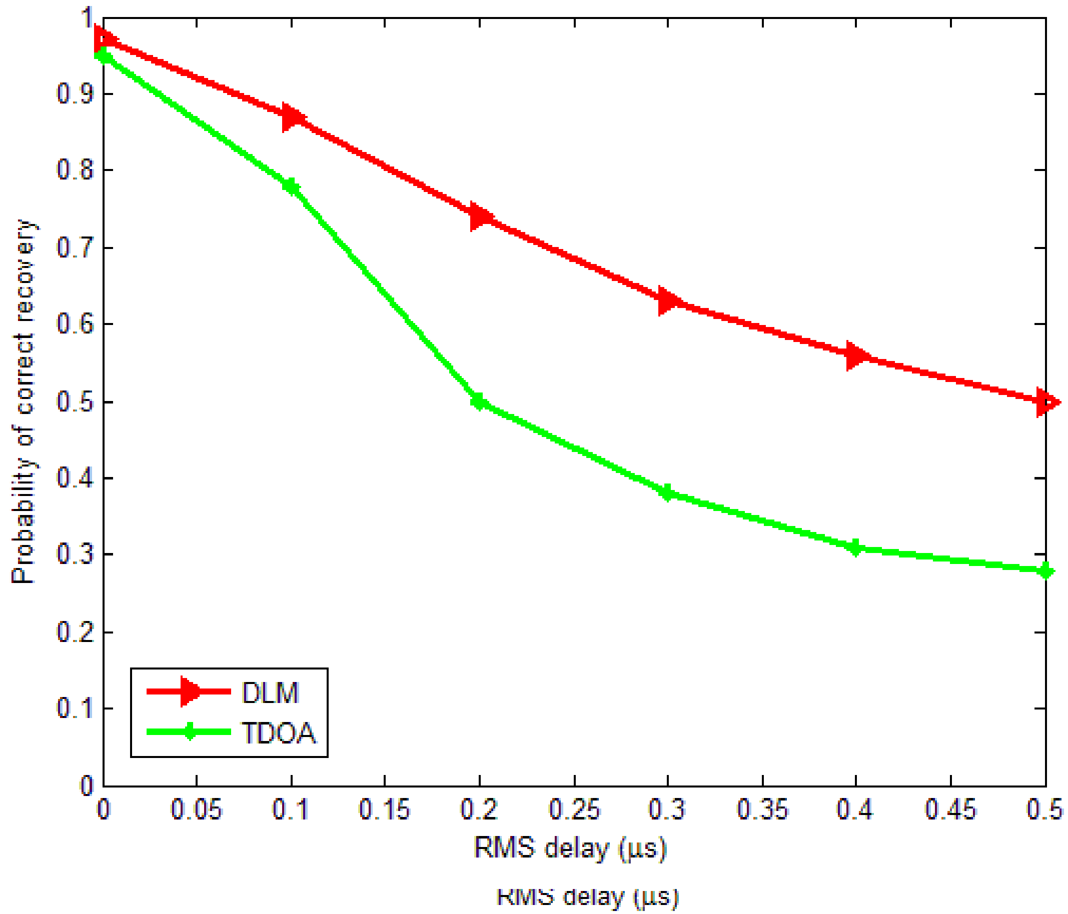
Probability of Correct Recovery vs. Error

- Error normalized to 30m
- SNR = 30 dB per sample (1000 samples and 5 sensors)



Probability of Correct Recovery vs. Delay Spread

- SNR = 30 dB per sample



Summary

- ✓ **A novel approach for localizing unknown emitters over multipath channels**
- ✓ **Solution developed directly from observations**
- ✓ **Solution relies on sparsity of emitters and of multipath**
- ✓ **Solution is blind with respect to transmitted signals and channel**
- ✓ **Mixed norm optimization exploits properties of LOS vs. NLOS**